Synchronization of Projective Frames

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Introduction

Synchronization of projective frames is a method of integrating sets of projectively reconstructed matrices in such a way that they differ from the true reconstruction by a single global projective transformation.

Projective Reconstruction

Assuming a pinhole camera model equation as,

$$\lambda_{ij}\, \mathbf{x_{ij}} = \mathbf{P_i}\, \mathbf{X_j} \quad (1)$$

Let two reconstructions be $\{P_a, P_b, P_c\}$ and $\{P'_a, P'_b, P'_d\}$ with unknown collineation T as,

$$\mathbf{P_i}\, \mathbf{T} \simeq \mathbf{P'_i} \quad (2)$$

After vector column-wise arrangement as,

$$\mathbf{vec}(\mathbf{P_i}\, \mathbf{T}) \simeq \mathbf{vec}(\mathbf{P'_i}) \quad (3)$$

Let a in $\mathbb{R}^n$ be $vec(P'_i)$, we have

$$\mathbf{B_i} = \begin{bmatrix} 0_{1\times(i-1)} & -a_{i+1} & a_i & 0 & 0 & \dots & 0 \\ 0_{1\times(i-1)} & -a_{i+2} & 0 & a_i & 0 & \dots & 0 \\ 0_{1\times(i-1)} & -a_{i+3} & 0 & 0 & a_i & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & & \dots \\ 0_{1\times(i-1)} & -a_n & 0 & 0 & 0 & \dots & a_i \end{bmatrix} \quad (4)$$

$$[\mathbf{a}]_\times = \begin{bmatrix} B_1 \\ B_2 \\ B_{n-1} \end{bmatrix} \quad (5)$$

$vec(P'_i)$ and $vec(P_i\, T)$ of $\mathbb{R}^n$ can be written as,

$$[\mathbf{vec}(\mathbf{P'_i})]_\times\, \mathbf{vec}(\mathbf{P_i}\, \mathbf{T}) = \mathbf{0} \quad (6)$$

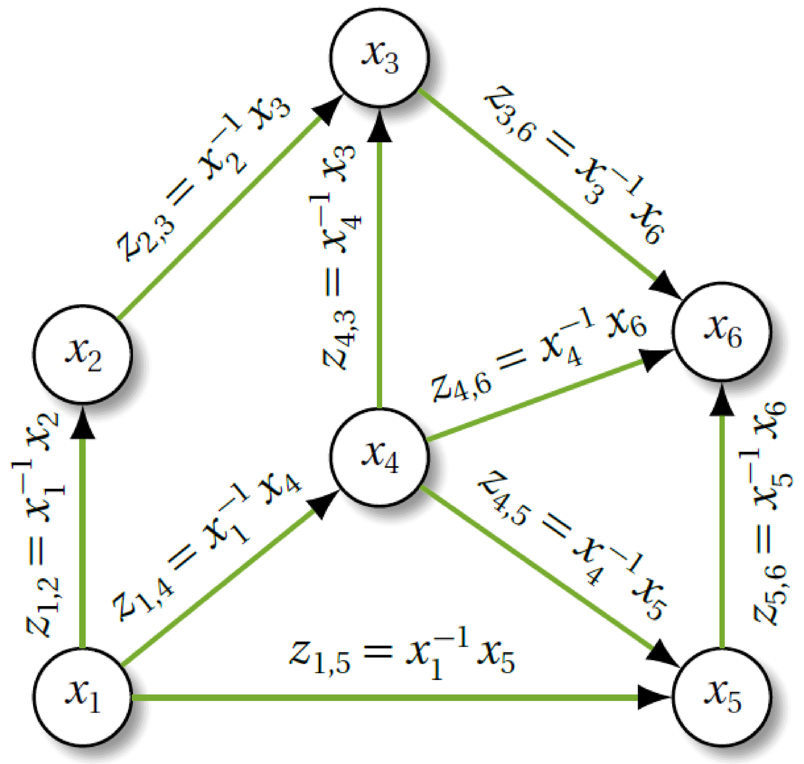
Using the properties of the Kronecker product,

$$[\mathbf{vec}(\mathbf{P'_i})]_\times\, (\mathbf{I_{4\times 4}} \otimes \mathbf{P_i})\, \mathbf{vec}(\mathbf{T}) = \mathbf{0} \quad (7)$$

Thus T is computed.

Synchronization

Let $\Gamma = (G, z)$ be a graph for $G = (V, E)$



For a graph, $U \in \mathbb{C}^{dn\times d}$ and $Z \in \mathbb{C}^{dn\times dn}$,

$$\mathbf{Z} = \mathbf{U}\mathbf{U^{-b}} \quad (8)$$

$$\mathbf{U} = \begin{bmatrix} X_1^{-1} \\ X_2^{-1} \\ \dots \\ X_n^{-1} \end{bmatrix}, \quad \mathbf{U^{-b}} = [X_1, X_2, \dots X_n,],$$

$$\mathbf{Z} = \begin{bmatrix} I & T_{1,2} & \dots & T_{1,n} \\ T_{2,1} & I & \dots & T_{2,n} \\ \dots & & & \dots \\ T_{n,1} & T_{n,2} & \dots & I \end{bmatrix} \quad (9)$$

With adjacency matrix A,

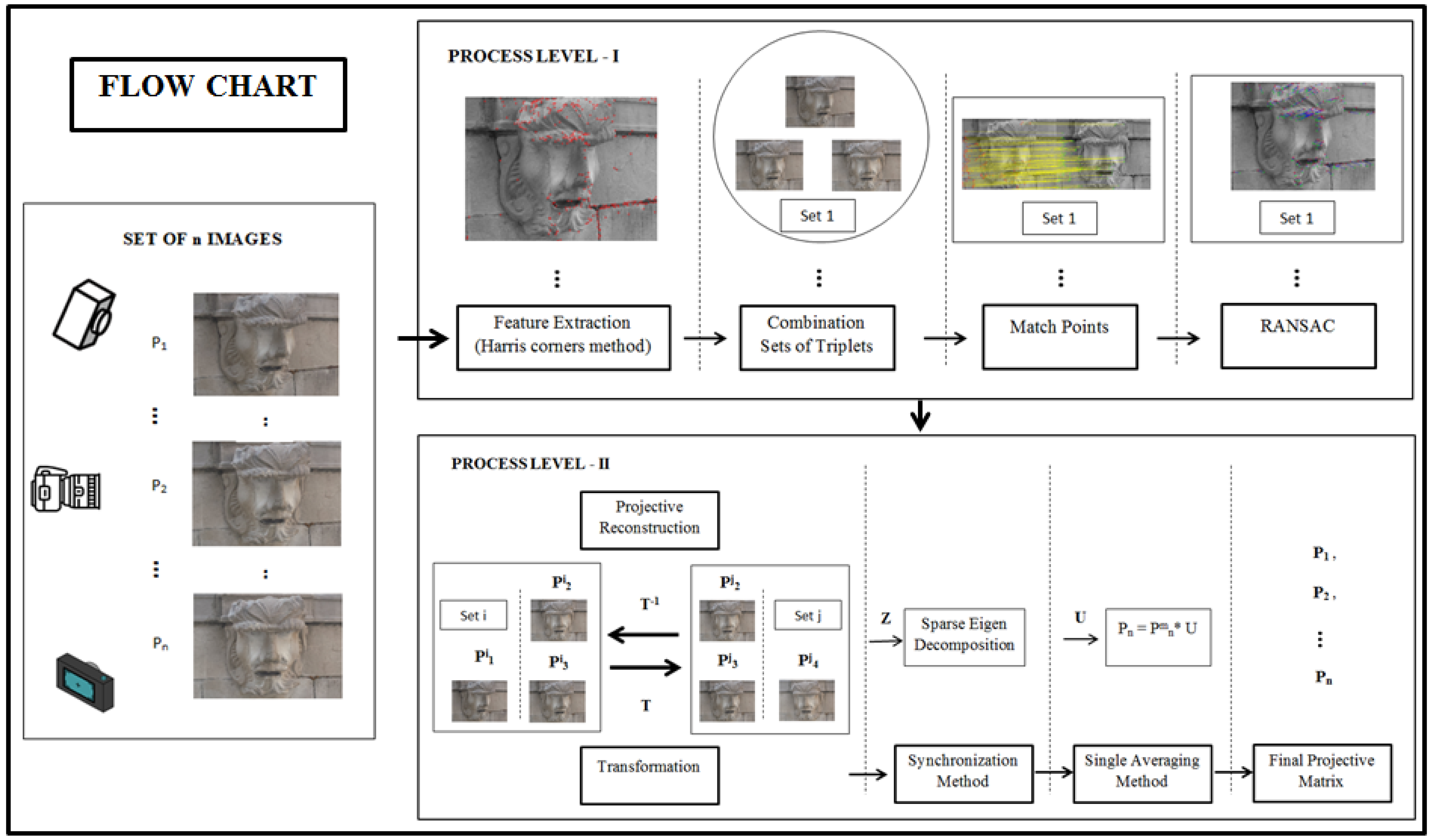
$$\mathbf{Z_A} = (\mathbf{U}\mathbf{U^{-b}}) \circ (\mathbf{A} \otimes \mathbb{1}_{\mathbf{d}\times\mathbf{d}}) \iff \mathbf{Z_A}\mathbf{U} = \mathbf{D}\mathbf{U} \quad (10)$$

where D sum of rows A and U is eigenvectors.

$$\mathbf{Z_A} = \begin{bmatrix} I/\zeta_1 & T_{1,2} & \dots & T_{1,n} \\ T_{2,1} & I/\zeta_2 & \dots & T_{2,n} \\ \dots & & & \dots \\ T_{n,1} & T_{n,2} & \dots & I/\zeta_n \end{bmatrix} \quad (11)$$

$$\mathbf{A_{i,j}} = \begin{cases} 1, & \text{if } \mathrm{T_{i,j}} \text{ known} \\ 0, & \text{otherwise} \end{cases}, \quad \zeta_{\mathbf{k}} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{A_{i,k}} \quad (12)$$

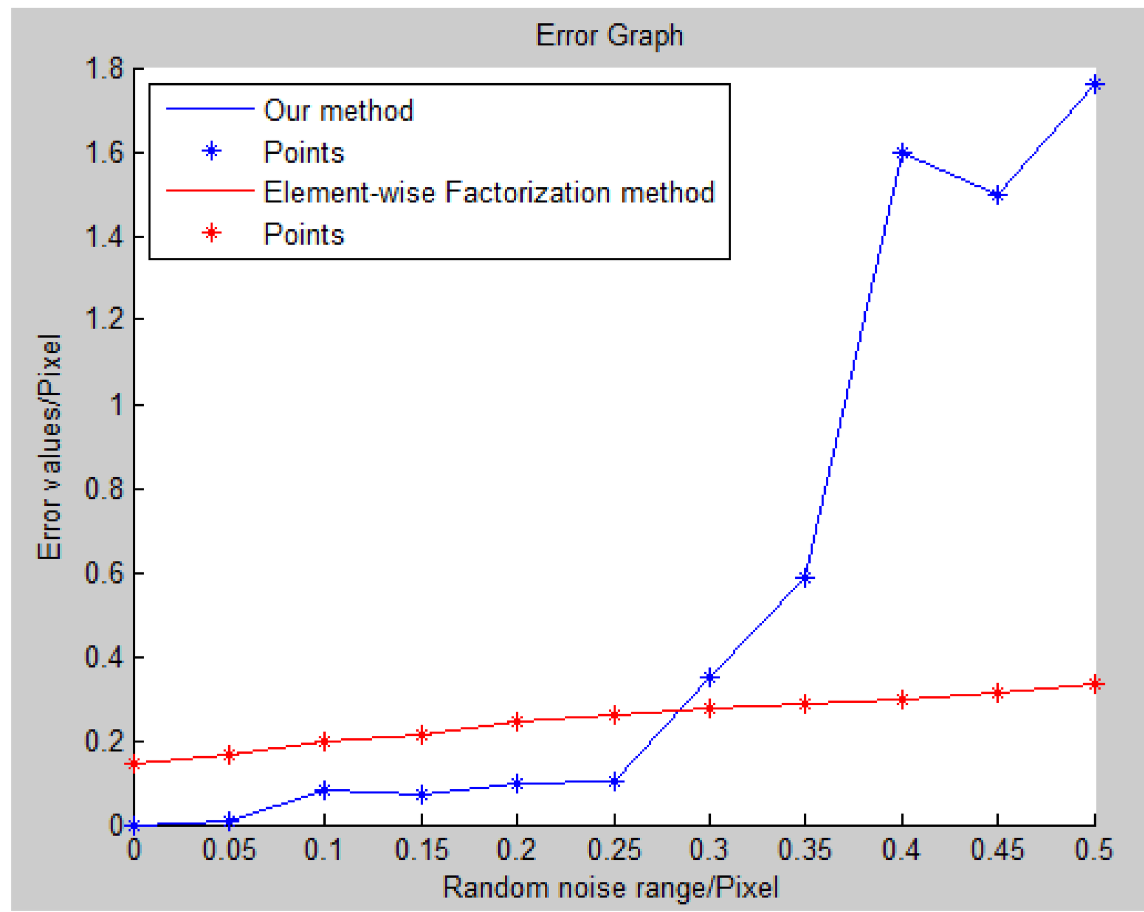
Flow Chart



Algorithm

- Feature points are extracted and grouped into combination of triplets using RANSAC.
- For sets of $\{x_i \leftrightarrow x'_i \leftrightarrow x''_i\}$ projective reconstruction P^i_j is done.
- The Transformation T is computed for global matrix Z.
- The synchronization method is used sparse Eigen decomposition, complex terms are removes using least square method.
- Compute the projective matrices after transformation. $\hat{P} = P^i_j * U$
- Single averaging method is used to get the final Projective matrices $\hat{P}_1, \hat{P}_2 ... \hat{P}_n$.

Result



References

[1] F Malapelle, A Fusiello, B Rossi, and E Piccinelli. Uncalibrated dynamic stereo using parallax.

[2] Federica Arrigoni, Andrea Fusiello, and Beatrice Rossi. Camera motion from group synchronization.

Future Research

The method based on tracking the 3D points from the final projective matrix, this way the relation between the image points and the projective matrices can be analyzed.

Explore the analogy between computing epipolar scales and the reconcilement of essential matrices, and try to extend it to fundamental matrices.

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